ECL 4340

POWER SYSTEMS

Lecture 7

Auto-Transformers, Tap-Changing Transformers, Regulating Transformers

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1

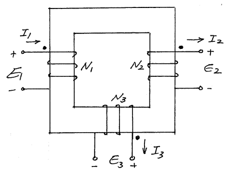
Announcement

- Please read Chapter 3
- HW #4 is due on September 23, Friday
- Midterm Exam on September 29, Thursday

2

Multi-Winding Transformers

Multi-Winding Transformers



Generalizing the two-winding transformer, additional windings can be used to form multi-winding transformers.

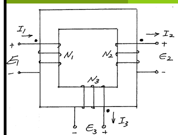
For three-winding transformers the mmf balance in the common flux path is

$$N_1 I_1 = N_2 I_2 + N_3 I_3$$

or $$I_1 = \frac{N_2}{N_1} I_2 + \frac{N_3}{N_1} I_3$$

3

Multi-Winding Transformers



$$N_1 I_1 = N_2 I_2 + N_3 I_3$$

or $$I_1 = \frac{N_2}{N_1} I_2 + \frac{N_3}{N_1} I_3$$

In per-unit system this becomes

(1) $$I_{1pu} = I_{2pu} + I_{3pu} \qquad \text{(Why?)}$$

For voltage, using the Faraday's law,

$$E_1 = j\omega N_1 \Phi, \quad E_2 = j\omega N_2 \Phi, \quad E_3 = j\omega N_3 \Phi$$

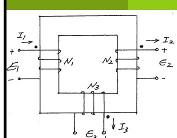
or $$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

Again, in per-unit system,

(2) $$E_{1pu} = E_{2pu} = E_{3pu} \qquad \text{(Why?)}$$

4

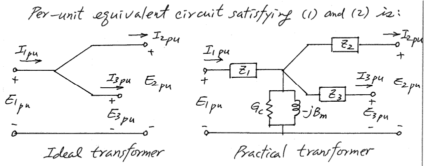
Multi-Winding Transformers



(1) $$I_{1pu} = I_{2pu} + I_{3pu}$$

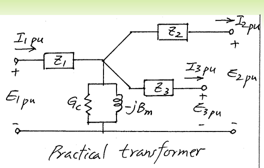
(2) $$E_{1pu} = E_{2pu} = E_{3pu}$$

Per-unit equivalent circuit satisfying (1) and (2) is:



5

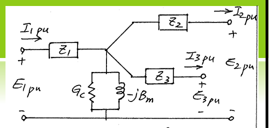
Multi-Winding Transformers



The shunt admittance can be determined by the open-circuit test and the series leakage impedances can be found by the short-circuit test as was done in two-winding transformer.

6

Multi-Winding Transformers



Practical transformer

Let Z_{ij} be the per-unit leakage impedance measured from winding i, with winding j shorted and winding k open, where $i, j, k = 1, 2, 3$.

Then

$$Z_{12} = Z_1 + Z_2$$
$$Z_{13} = Z_1 + Z_3$$
$$Z_{23} = Z_2 + Z_3$$

Solving these,

$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$$
$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$$
$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$$

7

Multi-Winding Transformers

Example 3.9: A 1ϕ tree-winding transformer:

N_1: 300 MVA, 13.8 kV
N_2: 300 MVA, 199.2 kV
N_3: 50 MVA, 19.92 kV

Leakage reactances, from short-circuit tests, are

$X_{12} = 0.1$ pu on 300 MVA, 13.8 kV base
$X_{13} = 0.16$ pu on 50 MVA, 13.8 kV base
$X_{23} = 0.14$ pu on 50 MVA, 199.2 kV base

Find the p.u. equivalent circuit using the base of 300 MVA and 13.8 kV on N_1.

8

Multi-Winding Transformers

Leakage reactances, from short-circuit tests, are

$X_{12} = 0.1$ pu on 300 MVA, 13.8 kV base
$X_{13} = 0.16$ pu on 50 MVA, 13.8 kV base
$X_{23} = 0.14$ pu on 50 MVA, 199.2 kV base

Find the p.u. equivalent circuit using the base of 300 MVA and 13.8 kV on N_1.

Solution: System base: $S_B = 300$ MVA
Base voltages: $V_{B1} = 13.8$ kV, $V_{B2} = 199.2$ kV, $V_{B3} = 19.92$ kV

New p.u. values:

$$X_{12} = 0.1 \text{ pu}$$
$$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96 \text{ pu}$$
$$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84 \text{ pu}$$

9

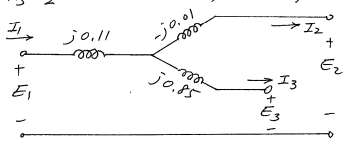
Multi-Winding Transformers

Reactances are

$X_1 = \frac{1}{2}(0.1 + 0.96 - 0.84) = 0.11\ pu$

$X_2 = \frac{1}{2}(0.1 + 0.84 - 0.96) = -0.01\ pu$

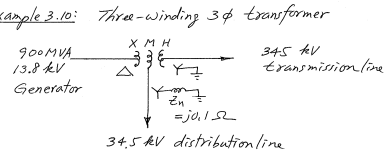
$X_3 = \frac{1}{2}(0.96 + 084 - 0.1) = 0.85\ pu$



10

Multi-Winding Transformers

Example 3.10: Three-winding 3ϕ transformer

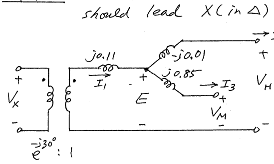


Draw the per-unit circuit using the 3ϕ base of generator as the system base.

11

Multi-Winding Transformers

Solution: In American Standard M and H (in Y) should lead X (in Δ) by 30°.



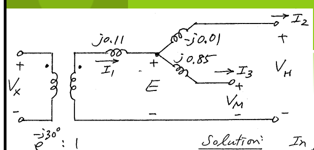
12

Multi-Winding Transformers

Example: Transmission line delivers 450 MVA at 345 kV with 0.8 pf lagging.
Resistive load of 150 MW at 34.5 kV in the distribution line.
Find the voltage at the generator terminal.

13

Multi-Winding Transformers



Solution: In per-unit, the transmission line current:

$$I_2 = \frac{450}{900}(0.8 - j0.6) = 0.4 - j0.3 = 0.5\angle -36.87° \text{ pu}$$

Resistance in distribution line is

$$R = (1.0)\left(\frac{900}{150}\right) = 6.0 \text{ pu}$$

Voltage at E due to the load current I_2:

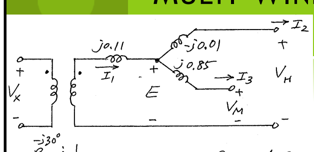
$$E = V_H + Z_2 I_2 = 1.0 + (-j0.01)(0.4 - j0.3)$$
$$= 1.0 - 0.003 - j0.004 = 0.997 - j0.004$$
$$\simeq 0.997\angle -0.23°$$

Current in the distribution line:

$$I_3 = \frac{E}{6 + j0.85} = \frac{0.997\angle -0.23°}{6.06\angle 8.06°} = 0.165\angle -8.29°$$

14

Multi-Winding Transformers



Current in the low voltage winding:

$$I_1 = I_2 + I_3 = 0.4 - j0.3 + (0.163 - j0.024)$$
$$= 0.563 - j0.324 = 0.65\angle -29.92°$$

Voltage at the generator terminal:

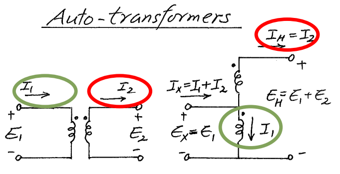
$$V_X = (e^{-j30°})(E + Z_1 I_1)$$
$$= (e^{-j30°})(0.997 - j0.004 + (j0.11)(0.563 - j0.324))$$
$$= (e^{-j30°})(\quad '' \quad + j0.062 + 0.0356)$$
$$= (e^{-j30°})(1.0326 + j0.058) = 1.0342\angle 3.215° - 30°$$
$$= 1.0342\angle -26.79°$$

$$|V_X| = (1.0342)(13.8 \text{ kV}) = \underline{\underline{14.27}} \text{ kV}$$

15

Auto- Transformers

Auto-transformers

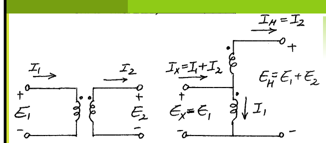


By connecting the two windings electrically we have a new transformer with increased voltage ratio.

NOTE: Current ratings should not exceed the original ratings of the windings.

16

Auto- Transformers



This will also increase power rating (in kVA).

For two-winding transformer the input & output powers are:

$$S_1 = E_1 I_1^*$$
$$S_2 = E_2 I_2^*$$

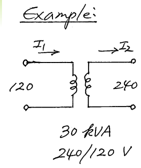
For auto-transformer the corresponding powers are:

$$S_X = E_1 (I_1 + I_2)^* = E_1 I_1^* + E_1 I_2^*$$
$$S_H = (E_2 + E_1)\, I_2^* = E_2 I_2^* + E_1 I_2^*$$

17

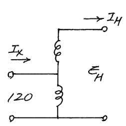
Auto- Transformers

Example:



Convert the 30 kVA 240/120 V transformer to an auto-transformer with 120 V as the low voltage.

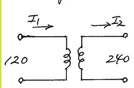
Find the new ratings of the auto-transformer.

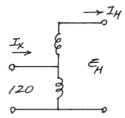


18

AUTO- TRANSFORMERS

Example:





30 kVA
240/120 V

Solution: Current ratings of windings are:

$I_1 = \frac{30,000}{120} = 250\,A$, $I_2 = \frac{30,000}{240} = 125\,A$

For auto-transformer the ratings are:

$E_X = 120\ V$ $\quad I_X = I_1 + I_2 = 375\ A$

$E_H = 120 + 240 = 360\ V$ $\quad I_H = 125\ A$

$S_X = 120 \times 375 = 45\ kVA$, $S_H = 360 \times 125 = 45\ kVA$

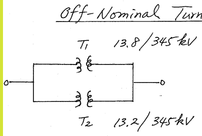
NOTE: Per-unit leakage impedance will decrease (Why ?)

Hint: $Z_{base\ old} = \frac{(240)^2}{30,000} = 1.92$, $Z_{base_{new}} = \frac{(360)^2}{45,000} = 2.88$

19

OFF-NOMINAL TURNS RATIO

Off-Nominal Turns Ratio



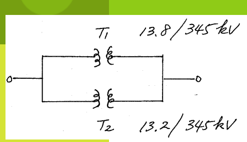
When two transformers are connected in parallel transmission lines, base voltages are supposed to be the same on each side of transformers.

However, this is not the case when the two transformers have different voltage ratings, due to:

1. not having identical transformers, or
2. one of the transformers changes taps.

20

OFF-NOMINAL TURNS RATIO



We choose the nominal voltages as base voltages:

$V_{base_1} = b\ V_{base_2}$,

while the rated voltages may be different:

$V_{1\,rated} = a_t\ V_{2\,rated}$, $\quad a_t$ = turns ratio, complex or real

To see how much a_t is different from the nominal voltage ratio b, take the ratio $c = \frac{a_t}{b}$ and rewrite above as

$V_{1\,rated} = b\left(\frac{a_t}{b}\right) V_{2\,rated} = b\,c\ V_{2\,rated}$

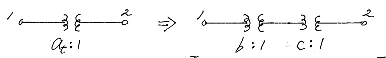
21

Off-Nominal Turns Ratio

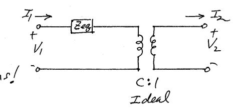
$V_{1\,rated} = a_t\, V_{2\,rated}$, a_t = turns ratio, complex or real

$$V_{1\,rated} = b\left(\frac{a_t}{b}\right) V_{2\,rated} = bc\; V_{2\,rated}$$

which can be viewed as two transformers in series:



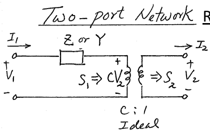
In per-unit only the first transformer disappears, while the second still remains!



22

Off-Nominal Turns Ratio

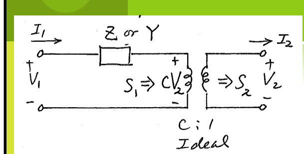
Two-port Network Representation



The off-nominal turns ratio (c) can be imbedded in a two-port network model using the admittance formulation (Y-parameters)

23

Off-Nominal Turns Ratio



Input and output of the ideal transformer are:

$$S_1 = (cV_2)\, I_1^* \;,\quad S_2 = V_2 I_2^*$$

Since $S_1 = S_2$, $\quad cV_2 I_1^* = V_2 I_2^* \;\Rightarrow\; I_2 = c^* I_1$

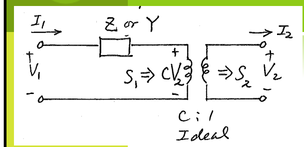
Input current is

$$I_1 = (V_1 - cV_2)\,Y$$
$$= YV_1 - cY\,V_2$$

and output current is $I_2 = c^* I_1 = c^* Y V_1 - |c|^2\, Y V_2$

24

Off-Nominal Turns Ratio



Input current is $I_1 = (V_1 - cV_2)Y$
$= YV_1 - cYV_2$

and output current is $I_2 = c^* I_1 = c^* Y V_1 - |c|^2 Y V_2$

Thus, the node equation in admittance formulation is

$$(1) \quad \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{array}{ll} Y_{11} = Y & Y_{12} = -cY \\ Y_{21} = -c^*Y & Y_{22} = |c|^2 Y \end{array}$$

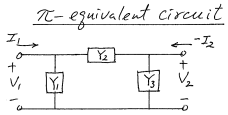
25

Off-Nominal Turns Ratio

Thus, the node equation in admittance formulation is

$$(1) \quad \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{array}{ll} Y_{11} = Y & Y_{12} = -cY \\ Y_{21} = -c^*Y & Y_{22} = |c|^2 Y \end{array}$$

π- equivalent circuit



If $c = c^*$, real, i.e., magnitude change, then
$Y_{12} = Y_{21}$, bi-lateral.

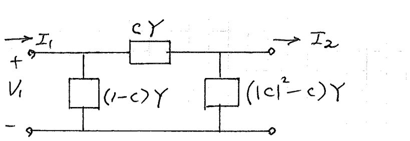
Thus, it can be modeled as a passive, bi-lateral two-port network.

$I_1 = Y_1V_1 + Y_2(V_1 - V_2)$
$= (Y_1 + Y_2)V_1 - Y_2V_2$
$-I_2 = Y_2(V_2 - V_1) + Y_3V_2$
$= -Y_2V_1 + (Y_2 + Y_3)V_2$
$\Rightarrow Y_{11} = Y_1 + Y_2, \quad Y_{12} = Y_{21} = -Y_2$
$Y_{22} = Y_2 + Y_3$

Comparing with (1) for c real,
$Y_2 = cY$
$Y_1 = Y - cY = (1-c)Y$
$Y_3 = (|c|^2 - c)Y$

26

Off-Nominal Turns Ratio



π-equivalent circuit for c real

27

Load Tap-Changing Transformers

- LTC transformers have tap ratios that can be varied to regulate bus voltages
- The typical range of variation is ±10% from the nominal values, usually in 33 discrete steps (0.0625% per step).
- Because tap changing is a mechanical process, LTC transformers usually have a 30 second deadband to avoid repeated changes.
- Unbalanced tap positions can cause "circulating vars"

28

Load Tap-Changing Transformers

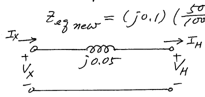
Example: 3ϕ transformer, 13.8 kVΔ/345 kVY, 1000 MVA
$Z_{eq} = j0.1$ pu, has high-voltage winding with ±10% taps.
Use system base: 500 MVA, 13.8 kV/345 kV
Find the per-unit equivalent circuit for:
a) Rated tap
b) −10% tap: 10% decrease in high-voltage side

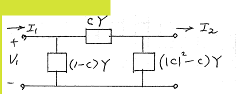
29

Load Tap-Changing Transformers

Example: 3ϕ transformer, 13.8 kVΔ/345 kVY, 1000 MVA
$Z_{eq} = j0.1$ pu, has high-voltage winding with ±10% taps.
Use system base: 500 MVA, 13.8 kV/345 kV
Solution:
a) For rated tap, $a_t = b = \frac{13.8}{345} = 0.04$, $c = 1$

$$Z_{eq\,new} = (j0.1)\left(\frac{500}{1000}\right) = j0.05 \text{ pu}$$





π-equivalent circuit for c real

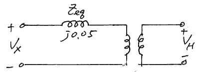
30

Load Tap-Changing Transformers

Example: 3ϕ transformer, 13.8 kV Δ/345 kV Y, 1000 MVA
$Z_{eq} = j0.1$ pu, has high-voltage winding with ± 10% taps.
Use system base: 500 MVA, 13.8 kV/345 kV

b) For −10% tap, $a_t = \frac{13.8}{345(1-0.1)} = \frac{b}{1-0.1} = 0.04444$

$c = \frac{a_t}{b} = \frac{1}{1-0.1} = 1.1111$



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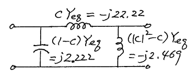


$c = \frac{1}{1-0.1} : 1$

or

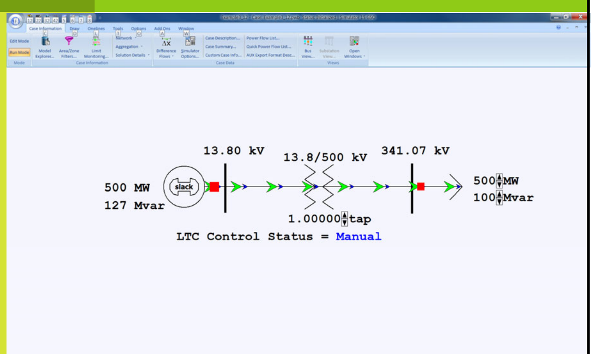
$1 : (1-0.1) = 0.9$

$Y_{eq} = \frac{1}{Z_{eq}} = \frac{1}{j0.05} = -j20$

NOTE: In general, the tap is modeled as $\boxed{1 : 1 + \Delta V}$

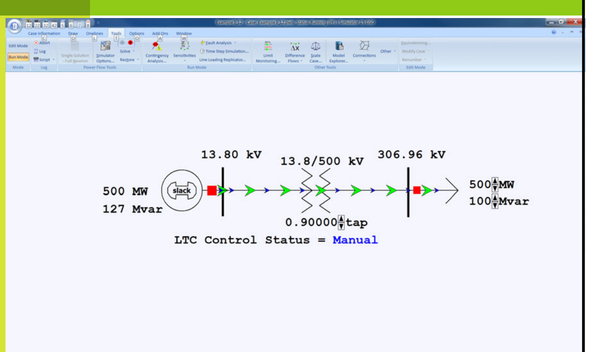
31

Load Tap-Changing Transformers



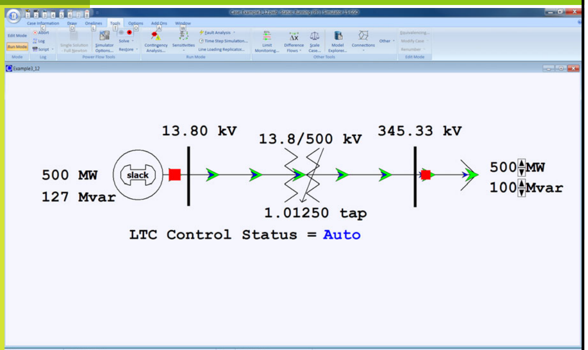
32

Load Tap-Changing Transformers



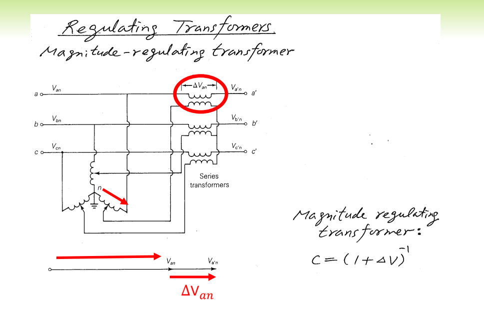
33

Load Tap-Changing Transformers



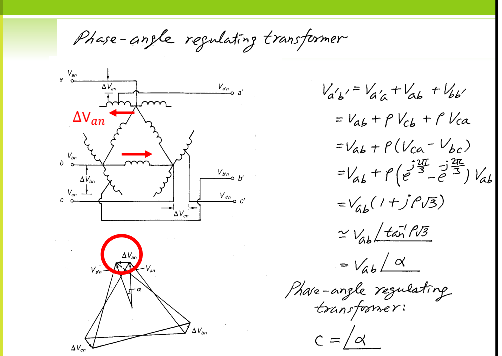
34

Regulating Transformers



35

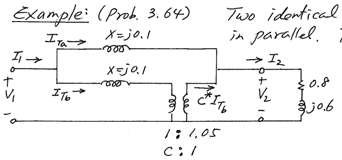
Regulating Transformers



36

Regulating Transformers

Example: (Prob. 3.64) Two identical transformers are in parallel. T_b has tap-setting at the load side.



Load : $0.8 + j0.6$ at $V_2 = 1.0\angle 0^\circ$ pu

a) Magnitude regulation : +5% increase in high-voltage winding

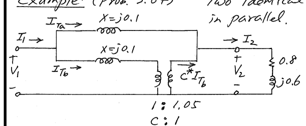
$$C = (1.05)^{-1} = 0.9524$$

Find current in each transformer and complex power delivered to the load through each transformer.

37

Regulating Transformers

Example: (Prob. 3.64) Two identical transformers are in parallel. T_b has tap-setting at the load side.



Load : $0.8 + j0.6$ at $V_2 = 1.0\angle 0^\circ$ pu

Node equations (Admittance formulation)

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where Y-parameters are for the two transformers in parallel.

Known variables:

$$V_2 = 1.0\angle 0^\circ, \quad I_2 = \frac{1.0\angle 0^\circ}{0.8 + j0.6} = 0.8 - j0.6$$

$\Rightarrow$

From the second equation

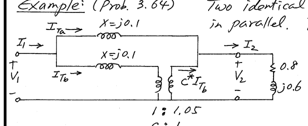
(1) $$-I_2 = Y_{21} V_1 + Y_{22} V_2$$

we can find V_1.

38

Regulating Transformers

Example: (Prob. 3.64) Two identical transformers are in parallel. T_b has tap-setting at the load side.



Load : $0.8 + j0.6$ at $V_2 = 1.0\angle 0^\circ$ pu

The admittance parameters are:

T_a: $Y_{11} = \frac{1}{j0.1} = -j10 = Y_{22}$

$Y_{12} = Y_{21} = -\frac{1}{j0.1} = j10$

T_b: $Y_{11} = \frac{1}{j0.1} = -j10, \quad Y_{22} = |C|^2 Y = (0.9524)^2 \left(\frac{1}{j0.1}\right) = -j9.07$

$Y_{12} = Y_{21} = -CY = -(0.9524)\left(\frac{1}{j0.1}\right) = j9.52$

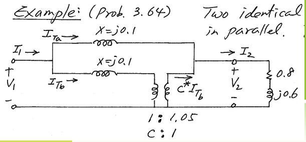
Parallel: $Y_{11} = -j10 - j10 = -j20, \quad Y_{22} = -j10 - j9.07 = -j19.07$

$Y_{12} = Y_{21} = j10 + j9.52 = j19.52$

39

Transformers

Example: (Prob. 3.64) Two identical in parallel.



$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From (1),

$$-(0.8 - j0.6) = (j19.52)V_1 + (-j19.07)(1\angle 0^\circ)$$

$$V_1 = 1.008 + j0.041$$

$\Rightarrow$ Current through each transformer:

$$I_{Ta} = \left(\frac{1}{j0.1}\right)(V_1 - V_2) = 0.41 - j0.08$$

$$c^* I_{Tb} = I_2 - I_{Ta} = 0.8 - j0.6 - (0.41 - j0.08)$$

$$= 0.39 - j0.52$$

$\Rightarrow$ Complex power through each transformer:

$$S_{Ta} = V_2 I_{Ta}^* = \underline{0.41 + j0.08}$$

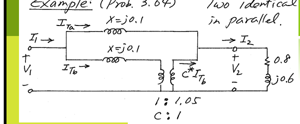
$$S_{Tb} = V_2 (c^* I_{Tb})^* = \underline{0.39 + j0.52}$$

NOTE: Reactive power flow is increased due to magnitude increase!

40

Regulating Transformers

Example: (Prob. 3.64) Two identical in parallel.



T_a: $Y_{11} = \frac{1}{j0.1} = -j10 = Y_{22}$

$Y_{12} = Y_{21} = -\frac{1}{j0.1} = j10$

b) Phase-angle regulation: $+3^\circ$ phase shift at the load $\Rightarrow c = 1\angle -3^\circ$

The admittance parameters:

T_b: $Y_{11} = \frac{1}{j0.1} = -j10$, $Y_{22} = |c|^2 Y = (1.0)^2(-j10) = -j10$

$Y_{12} = -cY = -(1.0\angle -3^\circ)(-j10) = 10\angle 87^\circ$

$Y_{21} = -c^* Y = -(1.0\angle +3^\circ)(-j10) = 10\angle 93^\circ$

T_a & T_b in parallel:

$Y_{11} = -j10 - j10 = -j20$, $Y_{22} = -j10 - j10 = -j20$

$Y_{12} = j10 + 10\angle 87^\circ = j10 + (0.5234 + j9.9863)$

$= 0.5234 + j19.986$

$Y_{21} = j10 + 1.0\angle 93^\circ = -0.5234 + j20$

41

Regulating Transformers

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From (1),

$$-(0.8 - j0.6) = (-0.523 + j20)V_1 + (-j20)(1.0\angle 0^\circ)$$

$$V_1 = 1.03 + j0.013$$

$$I_{Ta} = (V_1 - V_2)Y = (0.03 + j0.013)(-j10) = 0.13 - j0.30$$

$$c^* I_{Tb} = I_2 - I_{Ta} = 0.8 - j0.6 - (0.13 - j0.30) = 0.67 - j0.30$$

$$S_{Ta} = V_2 I_{Ta}^* = \underline{0.13 + j0.30}$$

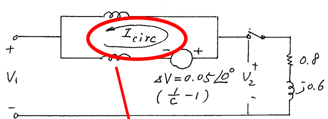
$$S_{Tb} = V_2 (c^* I_{Tb})^* = \underline{0.67 + j0.30}$$

NOTE: Real power is increased due to increase in phase angle!

42

LTC and Circulating Current

Example: Approximate by circulating current method



a) Magnitude regulation:

$$I_{circ} = \frac{0.05}{j0.2} = -j0.25 \text{ pu}$$

$$I_{Ta} = \frac{1}{2}(0.8 - j0.6) - (-j0.25) = 0.4 - j0.05$$

$$I_{Tb} = \frac{1}{2}(0.8 - j0.6) + (-j0.25) = 0.4 - j0.55$$

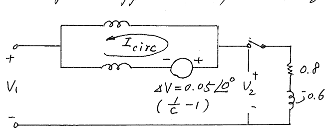
$$S_{Ta} = V_2 I_{Ta}^* = \underline{0.4 + j0.05}$$

$$S_{Tb} = V_2 I_{Tb}^* = \underline{0.4 + j0.55}$$

43

LTC and Circulating Current

Example: Approximate by circulating current method



b) Phase-angle regulation:

$$\Delta V = \frac{1}{c} - 1 = 1.0\angle 3^\circ - 1.0\angle 0^\circ = 0.0524\angle 91.5^\circ$$

$$I_{circ} = \frac{0.0524\angle 91.5^\circ}{j0.2} = 0.262 + j0.0069$$

$$I_{Ta} = 0.4 - j0.3 - (0.262 + j0.007) = 0.138 - j0.307$$

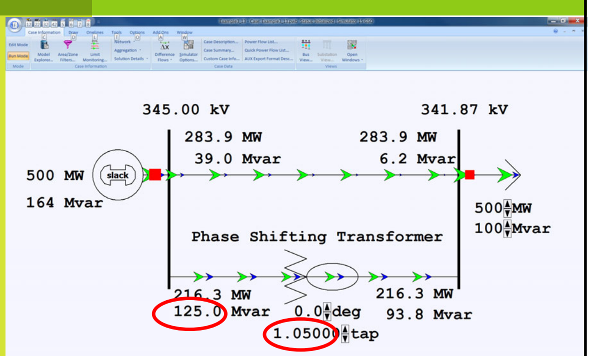
$$I_{Tb} = 0.4 - j0.3 + (0.262 + j0.007) = 0.662 - j0.293$$

$$S_{Ta} = \underline{0.138 + j0.307}$$

$$S_{Tb} = \underline{0.662 + j0.293}$$

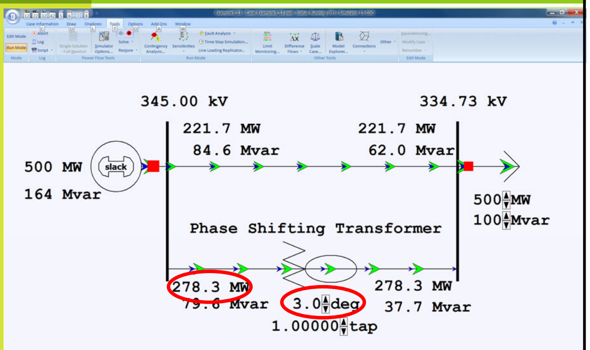
44

Phase Shifting Transformer



45

Phase Shifting Transformer



46